On the saturation of the lattice of z-ideals of pointfree function rings

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Throughout, by “ring" we mean a commutative ring with identity. For a ring A and $a \in A$, we let

$$\mathfrak{M}(a) = \{M \in \operatorname{Max} A \mid a \in M\}.$$

In the article

G. Mason, *z-Ideals and Prime Ideals*, J. Alg. 26 (1973), 280-297,

Mason calls an ideal I of A a *z-ideal* if for any a and b in A,

$$a \in I \quad \text{and} \quad \mathfrak{M}(a) = \mathfrak{M}(b) \quad \Rightarrow \quad b \in I.$$

Let L be a completely regular frame, and $\mathcal{R}L$ be the ring of real-valued continuous functions on L with $\mathcal{R}^*L$ as the subring of its bounded elements.

An ideal Q of $\mathcal{R}L$ is a *z-ideal* iff $Q = \bigcup\{\mathbf{M}_{\operatorname{coz}\alpha} \mid \alpha \in Q\}$,

where, for any $a \in L$,

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Recall that an ideal I of a ring A is said to be singular if it consists entirely of zero-divisors. For any $a \in A$, let P_a denote the intersection of all minimal prime ideals of A containing a. We will denote the annihilator of $\{a\}$ by $a^{\perp}$.

It is shown in

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that $P_a = a^{\perp\perp}$.

An ideal I of the ring A is called a *d*-ideal if $a^{\perp\perp} \subseteq I$, for every $a \in I$.

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- A frame is a complete lattice L in which the distributive law

$$a \wedge \bigvee S = \bigvee \{a \wedge x \mid x \in S\}$$

  holds for all $a \in L$ and $S \subseteq L$.

- Frame homomorphisms are maps that preserve the frame structure.

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- An element a of L is rather below an element b, written $a \prec b$, in case there is an element s, called a separating element, such that $a \wedge s = 0$ and $s \vee b = 1$.
- The frame L is regular if $a = \bigvee\{x \in L \mid x \prec a\}$ for each $a \in L$.
- An element a is completely below b, written $a \prec\!\prec b$, if there are elements (x_r) indexed by rational numbers $\mathbb{Q} \cap [0, 1]$ such that $a = x_0$, $x_1 = b$ and $x_r \prec x_s$ for $r < s$.
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- Coz L is the cozero part of L, and is the regular sub-σ-frame consisting of all the cozero elements of L.
- βL is the Stone-Čech compactification of L and it is the frame of regular ideals of Coz L. We denote by $j_L : \beta L \to L$ the join map $J \mapsto \bigvee J$, and the right adjoint of j_L is here denoted by r_L.
- For $I \in \beta L$,

$$\boldsymbol{M}^I = \{\alpha \in \mathcal{R}L \mid r_L(\mathrm{coz}\,\alpha) \subseteq I\} \quad \text{and} \quad \boldsymbol{O}^I = \{\alpha \in \mathcal{R}L \mid r_L(\mathrm{coz}\,\alpha) \prec I\}.$$

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Definition

- Let $Max(L)$ be the set of maximal elements of a frame L.
- An ideal of a ring A is called radical if it does not contain squares of non-members.
- A nucleus on a frame L is a closure operator $\ell : L \to L$ such that $\ell(a \wedge b) = \ell(a) \wedge \ell(b)$ for all $a, b \in L$.
- $\mathrm{Rad}(\mathcal{R}L)$ is the frame of radical ideals of $\mathcal{R}L$.
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Lemma

$\mathrm{Zid}(\mathcal{R}L) = \mathrm{Fix}(z)$, *for the z-nucleus on* $\mathrm{Rad}(\mathcal{R}L)$.

Proposition

$\mathfrak{K}(\mathrm{Zid}(\mathcal{R}L)) = \{M_{\mathrm{coz}\, a} \mid a \in \mathcal{R}L\}$.

Definition

- A frame L is *normal* if for any elements $a, b \in L$ such that $a \vee b = 1$, there are elements $c, d \in L$ such that $c \wedge d = 0$ and $a \vee c = 1 = b \vee d$.
- A frame L is *coherent* if it is *algebraic* and for all $a, b \in \mathfrak{K}(L)$, $a \wedge b \in \mathfrak{K}(L)$.

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- *$Zid(\mathcal{R}L)$ is a coherent frame.*
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In the article

Pointfree topology and the spectra of f-rings, *In: W.C. Holland, J. Martínez (eds.) Ordered Algebraic Structures, pp. 123–148. Kluwer, Deventer (1997).*

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Proposition

1. $\mathrm{Did}(\mathcal{R}L) = d(\mathrm{Zid}(\mathcal{R}L))$, *for the d-nucleus on* $\mathrm{Zid}(\mathcal{R}L)$.

   The d-nucleus on $\mathrm{Zid}(\mathcal{R}L)$ *takes the form*

$$d(Q) = \bigvee_{\mathrm{Zid}(\mathcal{R}L)} \{M_{(\mathrm{coz}\,\alpha)^{**}} \mid \alpha \in Q\} = \bigcup \{M_{(\mathrm{coz}\,\alpha)^{**}} \mid \alpha \in Q\}.$$

2. $\mathfrak{K}(\mathrm{Did}(\mathcal{R}L)) = \{M_{(\mathrm{coz}\,\alpha)^{**}} \mid \alpha \in \mathcal{R}L\} = \{M_{c^{**}} \mid c \in \mathrm{Coz}\,L\}$.
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Definition

Let L be a compact frame.

1. For each $a \in L$, consider $b \in L$ such that $b \vee y = 1$ implies $a \vee y = 1$. Let $s(a)$ denote the join of all such b. By a standard compactness argument, $s(a) \vee y = 1$ implies $a \vee y = 1$. Then $s(a)$ is called the saturation of a.
2. Let $s_L : L \to L$ denote the saturation nucleus on L and SL be the frame $\mathrm{Fix}(s_L)$.

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Lemma

Let $\{I_\lambda \mid \lambda \in \Lambda\}$ be a subset of βL. Then $\bigcap_\lambda \boldsymbol{M}^{I_\lambda} = \boldsymbol{M}^I$, where $I = \bigwedge_\lambda I_\lambda$.

Corollary

An ideal of $\mathcal{R}L$ is an intersection of maximal ideals iff it is of the form $\boldsymbol{M}^I$, for some $I \in \beta L$.

Lemma

For any $a, b \in L$, $\boldsymbol{M}_a \subseteq \boldsymbol{M}_b$ if and only if $a \leq b$.

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For any $a, b \in L$, $\boldsymbol{M}_a = \boldsymbol{M}_b$ if and only if $a = b$.

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It is known that for any coherent frame L and any $x \in L$

$$x = s(x) \quad \iff \quad x = \bigwedge\{m \in \mathrm{Max}(L) \mid x \leq m\}.$$

It follows therefore from the first Corollary above that the saturation of $\mathrm{Rad}(\mathcal{R}L)$ is

$$S\big(\mathrm{Rad}(\mathcal{R}L)\big) = \{\boldsymbol{M}^I \mid I \in \beta L\}.$$

Since $\mathrm{Rad}(\mathcal{R}L)$ and $\mathrm{Zid}(\mathcal{R}L)$ have exactly the same maximal elements, we deduce immediately that

$$S\big(\mathrm{Rad}(\mathcal{R}L)\big) = S\big(\mathrm{Zid}(\mathcal{R}L)\big).$$

We observe that the map $I \mapsto M^I$ is a frame homomorphism from βL into $S(\mathrm{Rad}(\mathcal{R}L))$ and this map is one-one. It is also clearly onto.

Proposition

$S(\mathrm{Zid}(\mathcal{R}L)) = S(\mathrm{Rad}(\mathcal{R}L)) \cong \beta L.$

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Proposition

$S(\text{Zid}(\mathcal{R}L)) = S(\text{Rad}(\mathcal{R}L)) \cong \beta L.$

- Recall that L is a quasi-F frame if and only if for all $a, b \in \mathrm{Coz}\, L$, $a \wedge b = 0$ and $a \vee b$ is dense, there exist $c, d \in \mathrm{Coz}\, L$ such that $a \wedge c = b \wedge d = 0$ and $c \vee d = 1$.
- A frame homomorphism $h\colon L \to M$ is flat if h is onto, and $h_*\colon M \to L$ is a lattice homomorphism.

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Proposition

If L is a completely regular quasi-F frame, then $\mathcal{S}(\mathrm{Did}(\mathcal{R}L)) \cong \beta L$.

Proof.

(OUTLINE) Since $\mathrm{Zid}(\mathcal{R}L)$ and $\mathrm{Did}(\mathcal{R}L)$ are compact normal frames, we only need to show that $d_l : \mathrm{Zid}(\mathcal{R}L) \to \mathrm{Did}(\mathcal{R}L)$ is flat.

- A frame L is a quasi-F iff βL is quasi-F.
- Since βL is spatial (modulo AC), $\beta L \cong \mathfrak{O}X$ for X equal to the spectrum of βL.
- If a space X is a quasi-F space, then the sum of two d-ideals in $C(X)$ is a d-ideal.
- $\mathcal{R}L$ is a ring of fractions of $\mathcal{R}^*L$, $I = I^{ce}$ for any ideal I of $\mathcal{R}L$.

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